The Clustering of Dark Matter

The Millennium Simulation Series

Simulations have enabled a full characterization of the (hierarchical) clustering of cold dark matter on large and small scales.

- Dark matter halos are self-similar in structure
- Mass function well constrained and understood

Dark matter halos in LCDM
The self-similar nature of LCDM halos

DM halos: self-similar structures linked by the age of the Universe

\[ M/R^3 = \text{constant} \]
The shape of the mass profiles of dark matter halos is roughly independent of halo mass and cosmological parameters.

Density profiles are “cerulean”

\[ \rho / \rho_{\text{crit}} = \delta_c / \left[ (r/r_s)(1+r/r_s)^2 \right] \]

At fixed mass, the only radial scale is set by the scale radius, \( r_s \)
The Acceleration Profile of Cold Dark Matter halos

- The $\rho \propto r^{-1}$ central cusp implies constant central acceleration: galaxies form in regions where the DM acceleration varies little with radius

$$a \propto GM(r)/r^2$$

$$a_{\text{max}} = a(0) = \text{const} \cdot V_{200}^2/r_{200}$$

$$a_{\text{max}}/(cH_0) \sim (1/10)(V_{200}/300 \text{ km/s})$$
The scaling laws of LCDM halos

Mass-Circular Velocity
\[ M_{200} \propto V_{200}^3 \]

Central acceleration:
\[ a_{\text{max}} = a(0) \sim V_{200}^2/r_{200} \]
\[ a_{\text{max}}/(cH_0) \sim 0.1(V_{200}/300 \text{ km/s}) \]

Mass-radius relation:
\[ M_{200} \propto r_s^3 \propto r_{200}^3 \]

Need a relation between galaxy mass, size, and halo mass, to interpret galaxy scaling laws.

\[ M_{200}/r_{200}^3 = \text{constant} \]
Galaxy Scaling Laws in LCDM

- Halo mass-galaxy mass relation
- Galaxy mass-size relation
- Tully-Fisher relation
- Mass discrepancy-acceleration relation
CDM halo mass function

[$n_{\text{in, M}} \left(\text{Mpc}/\text{h}\right)^{-3}$]

- CDM halo mass function is now well understood in all mass scales relevant to galaxy formation.

Schmidt et al 2009
CDM halo mass function vs galaxy luminosity function

- CDM halo mass function much steeper than the galaxy luminosity function at the faint end.
- Reconciling the two requires a highly non-linear dependence between galaxy and halo mass.
- At low halo masses reionization, as well as feedback from evolving stars, are thought to be responsible.
- Most dwarf galaxies live in halos of the same mass. Galaxy formation efficiency should decline in low mass halos.

Benson+2000
Abundance Matching: Galaxy Stellar Mass vs Halo Mass

- Galaxy formation efficiencies are very low, peak at 15% for Milky Way-like galaxies
- Galaxy masses are not simply proportional to halo masses
Large hydrodynamical simulations of cosmologically representative volumes (~100 Mpc box) have recently been completed (see; e.g., results from the Illustris Project).
Galaxy mass vs halo mass and size

- Galaxy mass-halo mass scaling is essentially that of abundance matching
- Galaxy mass-size relation of disc galaxies parallels the halo mass-size relation (Kravtsov+13)

Navarro+17
The Tully-Fisher relation
LCDM predicted circular velocity profiles

- CDM predicts a single mass/circular velocity profile for a given velocity scale.
- “Rotation curves” would be rising, not flat, without the contribution of baryons.
- $V_{\text{max}}$ and $r_{\text{max}}$ are another way of specifying the mass/size of the halo.

Oman+15
The Tully-Fisher relation

- A power-law scaling relating a disk galaxy’s luminosity (stellar mass) with its rotation speed

- A powerful secondary distance indicator

Recall that halos follow the relation: mass-circular velocity

\[ M_{200} \propto V_{200}^3 \]

**Velocity width** \( \sim 2 \times V_{\text{rot}} \)

Tully & Fisher 1977
The Tully-Fisher relation

- Zero-point and slope difficult to reproduce in early cosmological simulations

Navarro & Steinmetz 2000
The Tully-Fisher relation in LCDM

- TFR does not just reflect the mass-velocity ($M \propto V^3$) scaling of dark matter halos.
- Galaxy masses are not proportional to halo masses. Rotation velocities are a function of the galaxy mass, size, and the dark matter contribution within the galaxy half-mass radius.
- Need galaxies that are important gravitationally, and halos that “contract” as a result of galaxy assembly.
The Tully-Fisher relation in EAGLE

- Need galaxies that are important gravitationally (need to have the right size), and halos that “contract” as a result of the galaxy assembly
- At fixed halo mass, galaxies that are more massive than the average rotate faster than the average, and vice versa—the scatter spreads along the Tully-Fisher relation, leading to small dispersion in the relation

Ferrero+2016
The resulting relation is in excellent agreement with the observed Tully-Fisher relation, including its redshift evolution.
The Mass Discrepancy-Acceleration relation (MDAR)
The mass discrepancy-acceleration relation (MDAR)

\[ g_{\text{obs}} = \frac{V^2(R)}{R} \]

Disk galaxy rotation velocities may be “predicted” from the distribution of luminous matter

Bulge–Dominated Spiral (NGC7814)  |  Disk–Dominated Spiral (NGC6503)  |  Gas–Domained Dwarf (NGC3741)

Rotation Velocity [km s\(^{-1}\)]

Radius [kpc]

\[ g_{\text{bar}}(r) = GM_{\text{bar}}(<r)/r^2 \]

McGaugh+16 in PRL
The mass discrepancy-acceleration relation

\[ g_{\text{obs}} = \frac{V^2(R)}{R} \]

Two characteristic accelerations:

- \( a_0 \approx 10^{-10} \text{ m/s}^2 \): above which there is little need for dark matter, and
- \( a_{\text{min}} \approx 10^{-11} \text{ m/s}^2 \approx cH_0 \): a “minimum” acceleration probed by galaxies

For reference: Earth’s acceleration around the Sun is \( \approx 6 \times 10^{-3} \text{ m/s}^2 \)

\( cH_0 \) is \( \approx 7.2 \times 10^{-10} \text{ m/s}^2 \)

At the solar circle is \( \approx 2 \times 10^{-10} \text{ m/s}^2 \)

\[ g_{\text{bar}}(r) = GM_{\text{bar}}(<r)/r^2 \] (a proxy for surface brightness)
MDAR in LCDM

Navarro+17
The mass discrepancy-acceleration relation in EAGLE/APOSTLE

\[ g_{\text{bar}}(r) = GM_{\text{bar}}(<r)/r^2 \]
(a proxy for surface brightness)

Ludlow+16 in PRL
Navarro+18 in MNRAS
Summary

- The galaxy mass-halo mass relation follows closely simple abundance-matching relations.

- Galaxy sizes trace the scale radius of the halos they inhabit—they form in the “rising part” of the halo circular velocity curve.

- The Tully-Fisher relation results from the self-similar NFW halo structure and the mass-size relations indicated above.

- The mass discrepancy-acceleration relation follows naturally from the three earlier relations—a natural consequence of the self-similar structure of LCDM halos and the tight relations between galaxy mass, size, and halo mass.